Unit Circle

Radians and Degrees

$$2\pi = 360° \quad \pi = 180° \quad \frac{\pi}{2} = 90° \quad \frac{\pi}{4} = 45° \quad \frac{\pi}{6} = 30° \quad \frac{\pi}{3} = 60°$$

First Quadrant Trigonometry: Basic trigonometric functions on angles in the first quadrant. i.e. angles $\leq 90°\ or\ \frac{\pi}{2}$

$\cos(0) = 1$ $\quad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $\quad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ $\quad \cos\left(\frac{\pi}{2}\right) = 0$

$\sin(0) = 0$ $\quad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\quad \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ $\quad \sin\left(\frac{\pi}{2}\right) = 1$

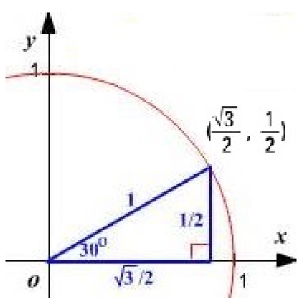
Once we know these we can find tangent by using $\tan\theta = \frac{\cos\theta}{\sin\theta}$. The unit circle can help us determine cosine and sine of many angles from 0 to 2π.

A **Unit Circle** is a circle, on a Cartesian coordinate system, that is centered at the origin and has radius equal to 1. Any point on the unit circle has $(x, y) = (\cos(\theta), \sin(\theta))$ where θ is the angle from the $x - axis$ to the line that extends from the origin to the point (x, y). This is due to the fact that the hypotenuse of any triangle made with the corresponding point (x, y) will be equal to 1.

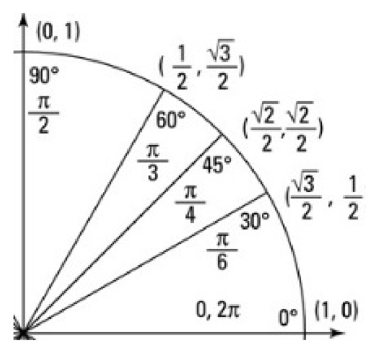
Example: The denominator for the cosine and sine are both = 1 ,

Thus the x-coordinate must be $= \frac{\sqrt{3}}{2}$ so that $\cos\left(\frac{\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$.

Thus we have $(x, y) = (\cos\left(\frac{\pi}{6}\right), \sin\left(\frac{\pi}{6}\right)) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$



First Quadrant: Thus, for the first quadrant we have the following picture of the main angles, and their corresponding cosines and sines:



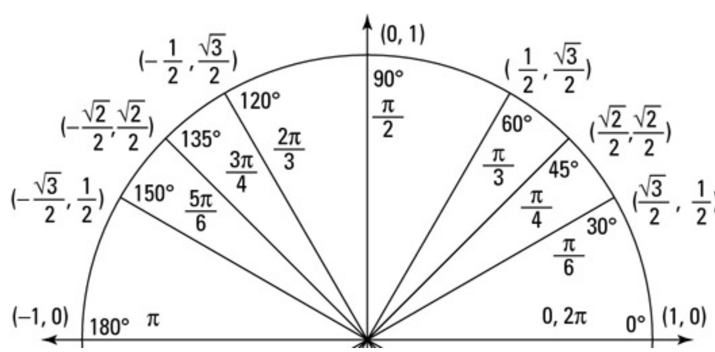
Second Quadrant: The cosine and the sine of the corresponding angles in the second quadrant are computed in the same manner, except the x-coordinate is going to the left, and thus the cosine is negative, and the y-coordinate is going up, and thus the sine is positive.

Example: The angle $\theta = \frac{5\pi}{6}$ creates the same triangle on the unit circle as the angle $\theta = \frac{\pi}{6}$ with the x-coordinate negative and the y-coordinate positive. Thus we have the following

$$\cos\left(\frac{5\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} \quad \text{and} \quad \sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Example: The angle $\theta = \frac{3\pi}{4}$ creates the same triangle on the unit circle as the angle $\theta = \frac{\pi}{4}$ with the x-coordinate negative and the y-coordinate positive. Thus we have the following:

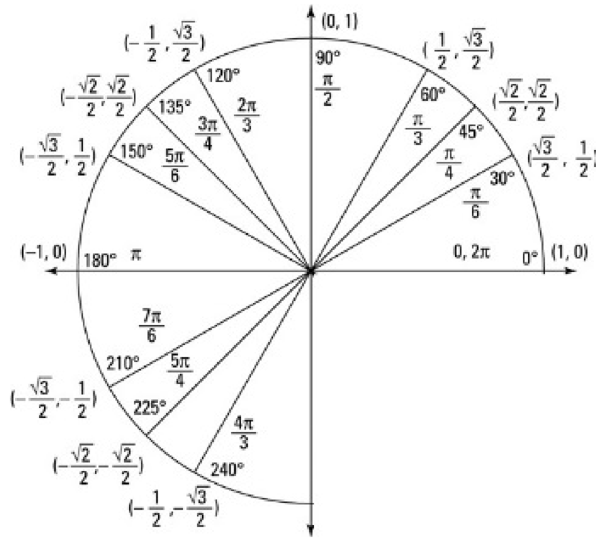
$$\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \quad \text{and} \quad \sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



Third Quadrant: The cosine and the sine of the corresponding angles in the third quadrant are computed in the same manner, except the x-coordinate is going to the left, and thus the cosine is negative, and the y-coordinate is going down, and thus the sine is negative.

Example: The angle $\theta = \frac{4\pi}{3}$ creates the same triangle on the unit circle as the angle $\theta = \frac{\pi}{3}$, except the x-coordinate is negative and the y-coordinate is negative. Thus we have the following:

$\cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$ and $\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

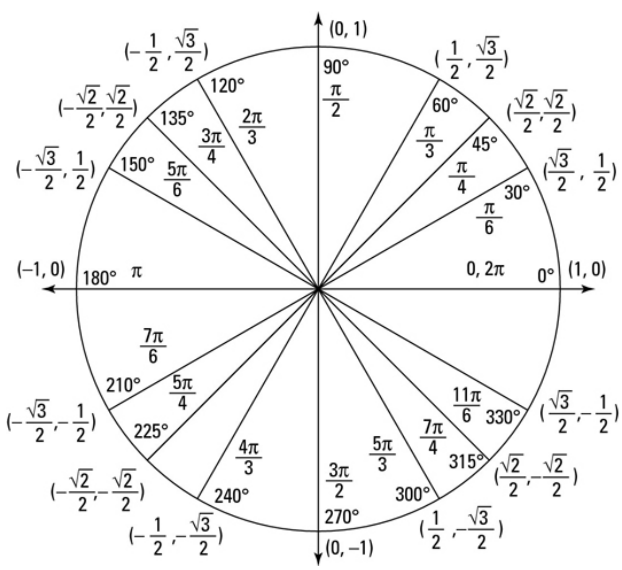


Fourth Quadrant: The cosine and the sine of the corresponding angles in the fourth quadrant are computed in the same manner, except the x-coordinate is going to the right, and thus the cosine is positive, and the y-coordinate is going down, and thus the sine is negative.

Example: The angle $\theta = \frac{7\pi}{4}$ creates the same triangle on the unit circle as the angle $= \frac{\pi}{4}$, except the x-coordinate is positive and the y-coordinate is negative. Thus we have the following

$\cos\left(\frac{7\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ and $\sin\left(\frac{7\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Complete Unit Circle of Trigonometry: Thus by remembering the first quadrant cosine and sine of these common angles, we can compute the cosine and sine of all the corresponding angles around the entire unit circle.



Unit Circle Graph: http://coremathseminars.com/home/trig__precalc

Cosine Graph: http://hotmath.com/hotmath_help/topics/cosine-function.html